Digital Vibration Sensor Using Delta-Sigma Modulation for Structural Health Monitoring

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ABSTRACT

A servo-type digital vibration sensor using delta-sigma modulation, which combines both advantages of feedback control system and delta-sigma modulation, has been proposed. The oversampled delta-sigma modulator is a noise shaping quantizer, which reshapes the spectrum of quantization noise so that most of the noise can be filtered out of the signal baseband. This inherently digital sensor is a highly accurate and flexible sensor. The signal-to-noise ratio of this sensor is determined by sampling frequency of the delta-sigma modulator. The cutoff-frequency of the lowpass filter in the feedback path is a controlling factor in digital sensor designing.

INTRODUCTION

Sensor technology plays a key role in structural health monitoring [1]. When the number of vibration sensors installed in structures increases significantly, the transfer and processing of a huge amount of data becomes a challenging task. However, this can be solved by developing digital vibration sensors interfaced with digital equipments for digital data processing and transmission [2]. Delta-sigma (Δ-Σ) modulation has become quite popular for achieving high resolution of analog to digital conversion [3], which can achieve over 20 effective number of bits (ENOB) resolution at reasonably high conversion speeds [4]. This paper will examine the noise shaping characteristics of delta-sigma modulation by means of theoretical analysis, numerical simulation and experimental verification, to propose a servo-type digital sensor using delta-sigma modulator. The simulation results exhibit good performance and high resolution.

DELTA-SIGMA MODULATION

Figure 1 shows a basic delta-sigma modulator used as analog-to-digital converter (ADC), which is a first-order feedback loop containing an internal low-resolution ADC and digital-to-analog converter (DAC), as well as a loop filter (here, an integrator).
It is a nonlinear system due to the quantizing effect of the ADC, as well as a dynamic one due to the memory in the integrator, and hence its analysis is a difficult mathematical task. However, simple quantitative understanding of its operation can be gained by using a discrete-time linearized model, as shown in Figure 2.

From the diagram,

\[ V(z) = Y(z) + E(z) = U(z) + (1 - z^{-1})E(z) \]  \hspace{1cm} (1)

Equation 1 can be written in the general form

\[ V(z) = STF(z)U(z) + NTF(z)E(z) \]  \hspace{1cm} (2)

Where in this case the signal transfer function (STF) is unity and the noise transfer function is expressed by

\[ NTF(z) = 1 - z^{-1} \]  \hspace{1cm} (3)

In order to estimate the in-band power of the quantization noise, by setting \( z = e^{j2\pi f} \), the squared magnitude of NTF in the frequency domain is defined by

\[ |NTF(e^{j2\pi f})|^2 = [2 \sin(\pi f)]^2 \]  \hspace{1cm} (4)
A simple way to construct a second-order delta-sigma modulator is to replace the quantizer within first-order delta-sigma modulator with another copy of first-order delta-sigma modulator. The resulting structure is shown in Figure 3.

![Figure 3. A z-domain linear model of second-order delta-sigma modulator](image)

From the diagram,

$$V(z) = U(z) + (1 - z^{-1})^2 E(z)$$  \hspace{1cm} (5)

Thus, for second-order delta-sigma modulator, the signal transfer function (STF) is unity and the noise transfer function is given by

$$NTF(z) = (1 - z^{-1})^2$$  \hspace{1cm} (6)

The squared magnitude of NTF in the frequency domain is

$$|NTF(e^{j2\pi f})|^2 = [2\sin(\pi f)]^4$$  \hspace{1cm} (7)

This is the square of that for first-order delta-sigma modulator. Hence we expect increased attenuation of quantization noise at low frequencies.

![Figure 4. Noise shaping function for (a) first-order and (b) second-order delta-sigma modulator](image)
Figure 4 (a) and (b) illustrate the frequency response of the NTF for first-order and second-order delta-sigma modulators, respectively. They exhibit clearly highpass response, which suppresses the quantization noise at and near dc (baseband) and amplifies it out of band, at and near 0.5 (i.e., $f_s/2$). This noise-shaping action is the key to the effectiveness of delta-sigma modulation.

Comparing the NTF magnitudes of first-order and second-order modulators at low frequencies, we can see that the former displays a $20\,\text{dB/decade}$ slope, while the latter has a $40\,\text{dB/decade}$ slope, as shown in Figure 5. The increased attenuation at frequencies close to dc is desirable because it reduces the amount of quantization noise within the signal baseband.

**SIMULATION OF DELTA-SIGMA MODULATORS**

Delta-sigma modulators use sampling rates much higher than the Nyquist rate, typically higher by a factor between 8 and 512, and generate each output utilizing all preceding input values. This property destroys the one-to-one relation between input and output samples. Now only a comparison of the complete input and output waveforms can be used to evaluate the converter’s accuracy, either in the time or in the frequency domain. Simulation is an important tool in delta-sigma modulator analysis because the linear model is imperfect and can hide important effects which can only become apparent when the true nonlinear nature of the modulation process is taken into account.

As a demonstration that delta-sigma modulation really does shape quantization noise, Figure 6 and 7 plot the spectrum of the output of a first-order delta-sigma modulator and a second-order delta-sigma modulator for oversampling ratio (the ratio of sampling rate of oversampled modulator to Nyquist-rate, i.e., twice of signal baseband) $OSR = 256$, respectively. These figures clearly show noise-shaping characteristics, also the $20\,\text{dB/decade}$ and $40\,\text{dB/decade}$ slopes of noise are consistent with first-order and second-order shaping as mentioned above.

A common measure of ADC’s accuracy is the signal-to-noise ratio (SNR) for a sine-wave input. The relationship between ENOB and SNR for an ideal Nyquist converter with sine-wave excitation is

$$SNR = 6.02 \times ENOB + 1.76$$ (8)
The inverse relationship of Equation 8 is often applied to oversampling converters to convert an SNR into an ENOB. Table I lists the SNR and ENOB values of first-order and second-order delta-sigma modulators for four different OSR. Note that for first-order delta-sigma modulator, doubling OSR increases the SNR by about 9 dB, and thus increases the ENOB by 1.5 bits, and for second-order case, the SNR increases by 15dB and the SNOB 2.5 bits for each doubling of the OSR.

| Table I. SNR AND ENOB OF DELTA-SIGMA ANALOG-DIGITAL CONVERTER |
|-------------------|--------|--------|--------|--------|
| SNR/ENOB          | OSR=64 | OSR=128 | OSR=256| OSR=512|
| 1st order ADC     | 45.4/7.3| 55.7/9.0| 65.1/10.5| 73.8/12.0|
| 2nd order ADC     | 69.4/11.2| 84.7/13.8| 99.9/16.3| 115.0/18.8|

EXPERIMENTAL VERIFICATION

An experiment was performed to confirm the characteristic of delta-sigma modulator. The experiment model adopts a second-order delta-sigma ADC (ADS1201 by Burr-Brown), which is followed by a sinc³ decimation filter (using Xilinx XC4010E FPGA), plotted in Figure 8. The sinc³ decimation filter here is used to reduce the sampling rate from \( F_s \) to \( F_d \).
Figure 8. Experiment block diagram

Figure 9 shows the PSD of output from the sinc\(^3\) decimation filter, clocked at \(F_d = 500\,\text{Hz}\), for two different sampling frequencies of ADC, 128\(kHz\) and 256\(kHz\). The resulting SNR values together with the theoretical values are given in Table II. Note that the existence of noises in addition to quantization noise, say harmonics of the signal (distinctly visible in Figure 9) which can’t be produced by white quantization noise, results in the difference between theoretical and experimental values.

![Figure 9. Experiment output spectrum](image)

<table>
<thead>
<tr>
<th>SNR</th>
<th>(Fs=128,\text{kHz})</th>
<th>(Fs=256,\text{kHz})</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EXPERIMENTAL</strong></td>
<td>70.2</td>
<td>72.0</td>
</tr>
<tr>
<td><strong>THEORETICAL</strong></td>
<td>101.0</td>
<td>112.3</td>
</tr>
</tbody>
</table>

**TABLE II. COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL VALUES**

**AN DIGITAL SENSOR USING \(\Delta-\Sigma\) ADC**

This part proposes a digital sensor using delta-sigma modulator, the schematic and block diagrams of which are plotted in Figure 10.

![Figure 10. (a) The schematic diagram and (b) block diagram of a digital sensor using delta-sigma ADC](image)
This is a servo-type digital sensor, combining both advantages of servo-type control system and delta-sigma modulation. In the forward path, the sensing element, which includes a pendulum, a displacement transducer, and an amplifier, is followed by a delta-sigma ADC. And in the feedback path, a DAC, here an analog lowpass filter, as well as a magnet-coil system producing feedback force, is required.

To verify the performance and accuracy of this digital sensor, Figure 11 plots the spectrum of the outputs using first-order and second-order delta-sigma modulators for three sampling frequencies. Furthermore, the resulting SNR and ENOB values are summarized in Table III. From the figure and table, we see that the digital sensor using second-order modulator has a higher PSD than that for the first-order modulator. For digital sensor, as much as 18 or higher bits resolution is required, thus the first-order ADC is inadequate for data conversion, which however can be performed by second-order ADC.

It is important to examine the effect of sampling frequency of delta-sigma modulator on PSD and SNR of digital output. In the numerical example, four different sampling frequencies, 128kHz, 256kHz, 512kHz, and 1024kHz, have been considered. Figure 11 and table III also illustrate that, either using first-order or second-order modulator, the digital output exhibits a higher SNR when the sampling frequency is increased.

![Figure 11. Output spectrum of digital sensor using (a) first-order modulator and (b) second-order modulator](image)

<table>
<thead>
<tr>
<th>SNR/ENOB</th>
<th>Fs=128 kHz</th>
<th>Fs=256 kHz</th>
<th>Fs=512 kHz</th>
<th>Fs=1024 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order ADC</td>
<td>31.6/5.0</td>
<td>44.9/7.2</td>
<td>54.6/8.8</td>
<td>71.9/11.6</td>
</tr>
<tr>
<td>2nd order ADC</td>
<td>63.6/10.3</td>
<td>78.8/12.8</td>
<td>100.5/16.4</td>
<td>110.7/18.1</td>
</tr>
</tbody>
</table>

It should be also pointed out that the cutoff frequency of lowpass filter in the feedback path, which acts as compensation for stability of the control system, is an important factor in digital sensor design. Here is a case of digital sensor using second-order delta-sigma modulator. In the simulation, a fourth-order Butterworth lowpass filter is adopted with varieties of cutoff frequencies. Table IV lists the SNR
values of sensor output at different cutoff frequencies of lowpass filter for three sampling frequencies of delta-sigma modulator.

It is instructive to find out the cutoff frequency of lowpass filter can only be set during a certain frequency region, for example, when \( F_s = 128 kHz \) the cutoff frequency should be from 2kHz to 8kHz, otherwise the control system will be unstable so that the reasonable output can’t be obtained. Also, it can be seen from Table IV that for a higher sampling frequency, a corresponding higher and wider frequency region should be considered for cutoff frequency.

<table>
<thead>
<tr>
<th>( F_s ) (kHz)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>–</td>
<td>62.8</td>
<td>60.1</td>
<td>64.2</td>
<td>63.6</td>
<td>65.3</td>
<td>60.3</td>
<td>60.6</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>256</td>
<td>–</td>
<td>79.7</td>
<td>79.3</td>
<td>78.9</td>
<td>78.8</td>
<td>81.2</td>
<td>78.7</td>
<td>83.1</td>
<td>84.4</td>
<td>78.8</td>
</tr>
<tr>
<td>512</td>
<td>–</td>
<td>93.4</td>
<td>92.6</td>
<td>91.8</td>
<td>100.5</td>
<td>94.5</td>
<td>92.2</td>
<td>92.2</td>
<td>95.3</td>
<td>94.4</td>
</tr>
</tbody>
</table>

Note: ‘–’ in the table denotes that the control system is unstable.

**CONCLUSIONS**

A digital vibration sensor using delta-sigma modulation has been proposed and simulated, with a variety of perspectives. By simulation, this digital sensor exhibits a good accuracy and flexibility because of the theoretically arbitrary high resolution of delta-sigma modulation and proportional relationship between SNR and sampling frequency. When designing this inherently digital sensor, only second or higher order delta-sigma modulator can be adopted as the first-order modulator is inadequate for resolution of data conversion for digital sensor. It is also pointed out that choosing the reasonable cutoff frequency of lowpass filter in the feedback path is also a considerable factor.

**REFERENCES**