Damage indicator defined as the distance between ARMA models for structural health monitoring

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SUMMARY

A new damage indicator denoted by the distance between ARMA models is proposed in this paper to identify structural damage including its location and severity. Two definitions are introduced as the distance, either the cepstral metric or subspace angles of ARMA models. However, the accuracy is deteriorated when the multiple inputs have strong correlations. To overcome this difficulty, a pre-whitening filter is applied. Thus, the proposed damage indicator is applicable for varieties of excitation types in civil engineering, such as wind, traffic loading and earthquake excitations. A five-storey building model is used for performance verification when subjected to different excitations. Ambient force and earthquake input have been used as excitations acting on the structure. Two calculating methods of the proposed damage indicators are both evaluated. When the excitations are mutually correlated, by using the pre-whitening filter, the damage identification ability of the proposed damage indicator improves significantly, especially for damage localization. The damage indicator increases monotonically with damage severity, which provides the potential for damage quantification. Copyright © 2007 John Wiley & Sons, Ltd.

KEY WORDS: damage indicator; ARMA model; cepstral metric; subspace angle; pre-whitening filter

1. INTRODUCTION

To seek a sensitive damage indicator is one of the keys to structural health monitoring (SHM) technology. An ideal damage indicator should give not only a qualitative indication whether damage might occur in the structure or not but also information about position and severity of the damage. To date, the majority of the literature on damage identification is based on the modal parameters, such as natural frequencies, modal shapes and so on that are proposed as the primary damage indicators [1–3]. Such modal-domain damage indicators have been most...
frequently investigated because the intuitive interpretation of modal parameters is possible. The modal parameters are global properties of the structure while damage is a local phenomenon, thus insensitive to local damage. Therefore, the insensitivity of the modal parameters due to local damage is the major obstacle to application of modal-domain damage indicators in practice.

Alternatively, the time-domain damage indicators based on time series analysis have been used to identify structural damage. Some researchers adopted the statistical pattern recognition methodology in which the single-input single-output linear time invariant (SISO LTI) model is used to interpret the time history of structural response and the residual of the autoregressive (AR) model or autoregressive model with exogenous inputs (ARX) model is considered as the damage indicator. For example, Fanning and Carden [4] used the mean and variance of the residuals of the AR model to form the statistical process control charts; Sohn and Farrar [5] and Mattson and Pandit [6] chose the standard deviation of the residual of AR–ARX model and vector AR (VAR) model as the damage indicator, respectively. Additionally, Nair et al. [7] proposed a sensitive damage indicator that takes only the first three AR coefficients of autoregressive moving-average (ARMA) model. These approaches have two main benefits: firstly, they do not require the mathematical model of structure, which is a time-consuming and difficult task especially for the complex and large structure; secondly, they are also output-only damage detection methods that are applied to the situation when the excitation is immeasurable or the controlled force excitation is impractical. However, these approaches are limited to qualitative identification or at most damage localization. They are unable to quantify the damage.

In this paper, a damage indicator defined as the distance between ARMA models is proposed. A reference ARMA model, which is fitted to the vibration response of the structure under normal operation conditions, represents the undamaged structure. If the structure is damaged, there exists a different model representing the damaged structure, and the distance between the two models may be correlated with the position and severity of the damage. For efficiently distinguishing between the damaged structure and the undamaged one, it is necessary to define an appropriate distance between ARMA models. The authors adopted the cepstral metric for ARMA models, which was proposed by Martin [8], to measure the distance between ARMA models. In fact, this metric is equivalently related to a common distance that is widely accepted in system identification for comparing ARMA models, the so-called subspace angles [9, 10]. Note that this cepstral distance is limited to SISO stochastic models or multiple-input multiple-output (MIMO) stochastic models with mutually independent input processes with the same variance. In civil engineering, however, the most excitations acting on structures are mutually dependent and correlated, such as wind traffic and earthquake. To overcome this difficulty, a pre-whitening filter is used for cancellation of spatial correlation of the excitations on different sites. Therefore, a damage detection methodology combining the cepstral damage indicator with a pre-whitening filter is proposed in this research.

The rest of the paper is organized as follows: Two equivalent approaches to calculate the distance between ARMA models using either cepstral metric or subspace angles are introduced in Section 2. The whitening transform of sensor signals for spatial decorrelation is presented in Section 3. Section 4 gives the simulations to verify the proposed damage detection methodology. A simulation model of five-storey building structure is used to evaluate the proposed
methodology, when subjected to ambient and earthquake excitations. Finally the conclusions are summarized in Section 5.

2. DISTANCE BETWEEN ARMA MODELS

An ARMA process is given by

\[ x_n = - \sum_{i=1}^{p} a_i x_{n-i} + \sum_{i=0}^{q} b_i e_{n-i} \]  

where \( a_i \) and \( b_i \) are the AR and MA coefficients, respectively. \( p \) and \( q \) are the model orders of the AR and MA processes, respectively, and \( e_n \) is a white noise process with zero mean and variance \( \sigma^2 \). The transfer function of a stable, minimum ARMA process has the following form in the \( z \)-domain:

\[
H(z) = \frac{\sum_{i=0}^{q} b_i z^{-i}}{\sum_{i=0}^{p} a_i z^{-i}} = \prod_{i=1}^{q} (1 - \beta_i z^{-1}) \prod_{i=1}^{p} (1 - \alpha_i z^{-1})
\]

where \( \alpha_i \) and \( \beta_i \) are the poles and zeros of the ARMA model, respectively.

The time series of ARMA process can be used to describe the dynamical response of a vibration structure. The parameters of the ARMA model are estimated to identify the natural frequencies and damping ratios \([11–13]\). Besides system identification, the ARMA estimation can also apparently be used for time series classification. For this purpose, the question is therefore how to compare two ARMA models, or equally, to find a metric to measure the distance between ARMA models. In this section, a cepstral metric for ARMA model is introduced, and its equivalent formation denoted by subspace angles between ARMA models is presented as well.

2.1. Cepstral metric for ARMA models

The cepstrum was firstly introduced by Bogert et al., who used it for the detection of echoes \([14]\). Cepstrum has been applied in a variety of areas including audio processing, speech processing, geophysics, medical imaging and others. To explore an application of the cepstrum in civil engineering, in this study, the cepstral distance between ARMA models \([8]\) is referred to as the damage indicator. This distance is expressed by a weighted Euclidean distance between cepstrums.

The power cepstrum of a SISO-LTI model with transfer function \( H(z) \) is the inverse Fourier transform of the logarithm of its power spectrum \( P(z) \) \([15]\):

\[
\log P(z) = \log H(z)H(z^{-1}) = \sum_{n \in z} C_n z^{-n}
\]

where \( C_n \) are the cepstrum coefficients.

Substituting (2) into (3), the logarithm of the power spectrum becomes

\[
\log P(z) = - \sum_{i=1}^{p} \log|z - \alpha_i|^2 + \sum_{i=1}^{q} \log|z - \beta_i|^2 + \log \sigma^2
\]
Then, the power cepstrum of the ARMA model can be expressed in terms of the poles and zeroes as follows:

\[
C_n = \begin{cases} 
\frac{1}{n} \left[ \sum_{i=1}^{p} a_i^n - \sum_{i=1}^{q} b_i^n \right], & n > 0 \\
\log \sigma^2, & n = 0 \\
\frac{1}{n} \left[ \sum_{i=1}^{p} \bar{a}_i^{-n} - \sum_{i=1}^{p} \bar{b}_i^{-n} \right], & n < 0
\end{cases}
\] (5)

Since the poles and zeroes occur in complex conjugate pairs, the above expression becomes

\[
C_n = \begin{cases} 
\frac{1}{|n|} \left[ \sum_{i=1}^{p} a_i^{|n|} - \sum_{i=1}^{q} b_i^{|n|} \right], & n \neq 0 \\
\log \sigma^2, & n = 0
\end{cases}
\] (6)

To measure the distance between ARMA models, a metric based on the cepstrum was proposed in [8]. For two ARMA models \(M^{(1)}\) and \(M^{(2)}\) with the associated cepstrum coefficients \(C_n^{(1)}\) and \(C_n^{(2)}\), the cepstral metric is defined as follows:

\[
D(M^{(1)}, M^{(2)}) = \sum_{n=1}^{\infty} n |C_n^{(1)} - C_n^{(2)}|^2
\]

\[
C_n = \frac{1}{|n|} \left[ \sum_{i=1}^{p} a_i^{|n|} - \sum_{i=1}^{q} b_i^{|n|} \right], \quad n \geq 1
\] (7)

As the cepstral metric is a Euclidean distance, the following property on the set of ARMA models holds

\[
D(M^{(1)} M^{(3)}, M^{(2)} M^{(3)}) = D(M^{(1)}, M^{(2)})
\] (8)

In other words, if two models \(M^{(1)}\) and \(M^{(2)}\) are passed through the same linear filter with model \(M^{(3)}\), their mutual distance is unaltered.

Therefore, if \(H_{\text{ARMA}}^{(1)}(z) = h^{(1)}(z)/d^{(1)}(z)\) and \(H_{\text{ARMA}}^{(2)}(z) = h^{(2)}(z)/d^{(2)}(z)\) are the transfer functions of two ARMA models \(M^{(1)}\) and \(M^{(2)}\), respectively, and assuming that the third ARMA model \(M^{(3)}\) has transfer function \(H^{(3)}(z) = 1/h^{(1)}(z)b^{(2)}(z)\), then two AR models \(N^{(1)}\) and \(N^{(2)}\) with transfer functions can be constructed as follows:

\[
H_{\text{AR}}^{(1)}(z) = H_{\text{ARMA}}^{(1)}(z) H^{(3)}(z) = \frac{1}{d^{(1)}(z)b^{(2)}(z)}
\]

\[
H_{\text{AR}}^{(2)}(z) = H_{\text{ARMA}}^{(2)}(z) H^{(3)}(z) = \frac{1}{d^{(2)}(z)b^{(1)}(z)}
\] (9)

According to the property (8), there is

\[
D(M^{(1)}, M^{(2)}) = D(N^{(1)}, N^{(2)})
\] (10)

That is to say, to measure the distance between ARMA models \(M^{(1)}\) and \(M^{(2)}\), it is sufficient to consider AR models \(N^{(1)}\) and \(N^{(2)}\) only. Therefore, for two stable AR models \(M^{(1)} M^{(2)}\) with order \(p^{(1)}, p^{(2)}\) and the associated poles \(z_i^{(1)}, z_i^{(2)}\), the cepstral metric can be simplified only in
terms of the poles of AR models as follows:

\[ D(M^{(1)}, M^{(2)})^2 = \log \prod_{i=1}^{p^{(1)}} \prod_{j=1}^{p^{(2)}} \left( 1 - z_i^{(1)} z_j^{(2)} \right) \prod_{i=1}^{p^{(1)}} \prod_{j=1}^{p^{(2)}} \left( 1 - z_i^{(2)} z_j^{(1)} \right) \frac{\prod_{i=1}^{p^{(1)}} \prod_{j=1}^{p^{(2)}} \left( 1 - z_i^{(2)} z_j^{(1)} \right)}{\prod_{i=1}^{p^{(1)}} \prod_{j=1}^{p^{(2)}} \left( 1 - z_i^{(1)} z_j^{(2)} \right)} \]  

(11)

2.2. Subspace angles between ARMA models

A SISO-LTI stable, minimum phase ARMA model can also be described in the forward innovation state–space form:

\[
\begin{align*}
    x(k+1) &= Ax(k) + Ku(k) \\
    y(k) &= Cx(k) + u(k)
\end{align*}
\]

where \( y(k) \) is the output of the model, \( u(k) \) is the innovation process of \( y(k) \) and \( x(k) \) is the state process. The matrix \( A \) is called the system matrix, \( C \) is the output matrix and \( K \) is the Kalman gain. For the sake of brevity, the state–space model (12) is denoted by the threesome \( (A, K, C) \), the poles of which are the eigenvalues of the system matrix \( A \). The associated infinite observability matrix of the state–space model is expressed as

\[ O_\infty = [C \ A \ CA^2 \ \ldots ]^T \]  

(13)

From the above model (12), the state–space equations of the inverse model can be derived as follows:

\[
\begin{align*}
    x(k+1) &= (A - KC)x(k) + Ky(k) \\
    u(k) &= -Cx(k) + y(k)
\end{align*}
\]

(14)

Consequently, the zeroes of the model \( (A, K, C) \) are equal to the eigenvalues of \( (A - KC) \). Similarly, the infinite observability matrix of the inverse state–space model is denoted by

\[ O_{\infty} = [-C - C(A - KC) - C(A - KC)^2 \ \ldots ]^T \]  

(15)

Assume that \( M^{(1)} \) and \( M^{(2)} \) are two stable, minimum-phase ARMA models of order \( n \). \( O_{\infty}^{(1)} \) and \( O_{\infty}^{(2)} \) are the observability matrices of \( M^{(1)} \) and \( M^{(2)} \), respectively, and \( O_{\infty}^{(1)} \) and \( O_{\infty}^{(2)} \) are the observability matrices of the corresponding inverse models, respectively. The subspace angles between ARMA models \( M^{(1)} \) and \( M^{(2)} \) were defined as the principal angles \( \theta_i \) \( (i = 1, \ldots, 2n) \) between the subspace ranges \( (O_{\infty}^{(1)}, O_{\infty}^{(2)}) \) and \( (O_{\infty}^{(1)}, O_{\infty}^{(2)}) \) \( [10] \).

Note that the cepstral metric of ARMA models defined in \( [8] \) can be equivalently related to the subspace angles between ARMA models. For two ARMA models \( M^{(1)} \) and \( M^{(2)} \) of order \( n \), another expression of distance between ARMA models is defined in terms of subspace angles as follows:

\[ D(M^{(1)}, M^{(2)})^2 = \log \prod_{i=1}^{2n} \frac{1}{\cos^2 \theta_i} \]  

(16)

where \( \theta_i \) is the subspace angles between ARMA models \( M^{(1)} \) and \( M^{(2)} \).

Similarly to the cepstral metric only considering AR poles, the metric in terms of subspace angles between AR models can be derived. Assume that two stable and observable AR models \( M^{(1)} \) and \( M^{(2)} \) are characterized in state–space form by their system matrices \( A^{(1)} \) and \( A^{(2)} \) and output matrices \( C^{(1)} \) and \( C^{(2)} \), respectively. The associated infinite observability matrices are denoted by \( O_{\infty}^{(1)} \) and \( O_{\infty}^{(2)} \), respectively. Then, for two AR models \( M^{(1)} \) of order \( n^{(1)} \) and \( M^{(2)} \) of
order $n^{(2)}$, the metric in terms of subspace angles between AR models is equal to

$$D(M^{(1)}, M^{(2)})^2 = \log \prod_{i=1}^{n} \frac{1}{\cos^2 \theta_i}$$

(17)

where $n = \max(n^{(1)}, n^{(2)})$, and $\theta_i (i = 1, \ldots, n)$ are the subspace angles between AR models $M^{(1)}$ and $M^{(2)}$, which is defined as the principal angles between the subspace ranges $O_i^{(1)}$ and $O_i^{(2)}$.

3. PRE-WHITENING FILTER

Since the distance between ARMA models in terms of either cepstral metric (Section 2.1) or subspace angle (Section 2.2) is implicitly dependent on the input covariance matrix, the damage indicator denoted by this distance is limited to SISO stochastic models or MIMO stochastic models with mutually independent input processes with the same variance. In practice, however, the excitations acting on structures are mutually dependent and correlated, such as wind and traffic loading on bridge and inertial force on every storey of the building induced by earthquake. Therefore, this damage indicator cannot be directly applied to damage detection because of the correlation of multiple inputs. To overcome this difficulty, a pre-whitening filter is used for signal pre-processing before calculating the damage indicators.

The decorrelation techniques play important roles in signal processing. They are the basis for modern subspace methods of spectrum analysis and array processing and are often used in a preprocessing stage to eliminate redundancy or to reduce noise [16]. In this section a whitening transform technique, the symmetric whitening algorithm, is introduced. This signal preprocessing technology is applied to pre-whiten (decorrelate) the sensor signals from vibration structure before conducting the proposed damage detection methodology.

In whitening, the $m$-dimension sensor signals $x(k)$ are pre-processed by using the following whitening transformation:

$$y(k) = Wx(k)$$

(18)

where $y(k)$ denotes the whitened signals, and $W$ is the $m \times m$ whitening matrix. For this purpose, the matrix $W$ is chosen so that the covariance matrix $E[y(k)y(k)^T]$ becomes the unit matrix $I_m$. Thus the components of the whitened signals $y(k)$ are mutually uncorrelated and they have unit variance, i.e.

$$R_{yy} = E[y(k)y(k)^T] = E(Wxx^TW^T) = WR_{xx}W^T = I_m$$

(19)

In general, the sensor signals $x(k)$ are mutually correlated, i.e. the covariance matrix $R_{xx}$ is a full (not diagonal) matrix. It should be noted that the matrix $W$ is not unique, since by multiplying an arbitrary orthogonal matrix to $W$ from the left, a new $W$ is generated, and equality (19) is still preserved.

Usually, the covariance matrix sensor signals $x(k)$ is symmetric positive definite; it therefore can be decomposed as follows:

$$R_{xx} = V_{x} \Lambda_{x}^{1/2} \Lambda_{x}^{1/2} V_{x}^T$$

(20)

where $V_{x}$ is an orthogonal matrix and $\Lambda_{x} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)$ is a diagonal matrix with positive eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_m > 0$. Hence, under the condition that the covariance
matrix is positive definite, the required decorrelation matrix can be computed as follows:

\[
W = \Lambda^{-1/2}_x V^T_x = \text{diag}\left\{ \frac{1}{\sqrt{\lambda_1}}, \frac{1}{\sqrt{\lambda_2}}, \ldots, \frac{1}{\sqrt{\lambda_m}} \right\} V^T_x
\]

or

\[
W = U\Lambda^{-1/2}_x V^T_x
\]

where \(U\) is an arbitrary orthogonal matrix.

By substituting (20) and (21) or (22) into (19), it can be easily verified that

\[
R_{yy} = E\{yy^T\} = \Lambda^{-1/2}_x V^T_x A_x V^T_x V^T_x \Lambda^{-1/2}_x = I_m
\]

\[
R_{yy} = U\Lambda^{-1/2}_x V^T_x A_x V^T_x V^T_x \Lambda^{-1/2}_x U^T = I_m
\]

4. PERFORMANCE OF PROPOSED METHOD

In order to demonstrate the performance of the proposed damage indicator, simulation studies have been performed on a five-storey shear building structure model, which is depicted in Figure 1. This model can be simplified as a five degree-of-freedom (DOF) structure system. It is assumed in this case that the mass of every storey is 100 kg, and the lateral stiffness is 1 MN/m. The damping ratio is assumed to be 3% for all modes. The building structure is assumed to be subjected to ambient force on every storey or base excitation of various earthquake records. The inter-storey stiffness reduction ratio is assumed to be the structural damage. For evaluating the sensitivity of the proposed damage indicator, minor damage severities are assumed, which are 2, 4, 6, 8 and 10% stiffness reduction. Single damage cases occur on each storey, respectively, which are ‘one-storey damage’, ‘two-storey damage’,…, and ‘five-storey damage’. The inter-storey stiffness reduction between the first storey and the base is called as ‘1-storey damage’, and the

![Figure 1. Five-storey building and its simplified five DOF system.](image)
rest may be deduced by analogy. Hence, there are a total of 25 damage scenarios. In this research, the modal superposition algorithm is applied to simulate the dynamical responses of the structure system. The measurement noise is assumed to be Gaussian white noise process added to the sensor output. Every acceleration response has been added by the measurement noise of 10% noise level, which is the ratio of root mean square (RMS) of sensor noises to acceleration responses.

The flowchart of the damage detection methodology using the proposed damage indicator is shown in Figure 2. The first step is the pre-processing of the sensor signals from the vibration structure, typically acceleration on every storey of structure. By using the pre-whitening filter, the sensor signals, which are measured from each storey with mutually correlation, are decorrelated so that they have unit variance. In the next step, the uncorrelated signals are fitted to several ARMA models representing every DOF of the structure system, respectively. The reference ARMA models, which are fitted to the vibration signals from the structure under normal operation conditions, represent the undamaged structure, while the new ARMA models are fitted to the vibration signals from the damaged structure. Then the damage indicators are computed as the distance between ARMA models representing the undamaged and damaged structures. This distance can be calculated by either cepstral metric of ARMA models using poles and zeroes (i.e. Equation (7) or (11)), or subspace angles between ARMA models (i.e. Equation (16) or (17)). Finally, damage identification can be conducted according to the magnitude of the damage indicator on different sites, which implicate the location and severity of damages.

4.1. Ambient force

Firstly, the excitations are assumed to be ambient forces acting on every storey. The ambient forces are modeled as filtered Gaussian white noise processes, which approximate wind or other ambient excitations. The acceleration responses including measurement noise for five storeys are calculated as depicted in Figure 3.
Figure 3. Time series of simulated acceleration induced by ambient forces.

Figure 4. Damage indicators in terms of AR poles subjected to mutually independent ambient inputs: (a) one-storey; (b) two-storey; (c) three-storey; (d) four-storey; and (e) five-storey damage cases.
Figure 5. Damage indicators in terms of subspace angles subjected to mutually independent ambient inputs: (a) one-storey; (b) two-storey; (c) three-storey; (d) four-storey; and (e) five-storey damage cases.

Figure 6. Damage indicators without using pre-whitening filter subjected to mutually dependent ambient inputs: (a) one-storey; (b) two-storey; (c) three-storey; (d) four-storey; and (e) five-storey damage cases.
Case I. Mutually independent inputs: In case I, the excitations are modeled as five mutually independent filtered Gaussian white noise processes. Since the excitations on every storey are mutually independent of each other, the covariance matrix of the five acceleration responses is near to a diagonal matrix. Hence, there is no need to use the pre-whitening (first step in the flowchart) in this case, and the simulated original sensor signals are directly applied to compute the damage indicators.

For different damage scenarios, 25 corresponding damage indicators are calculated, which are shown in Figures 4 and 5. The two methods to compute the distance between ARMA models, introduced above, are both evaluated. Figure 4 shows the cepstral metrics calculated by using (11) in terms of the poles of AR models, and Figure 5 shows those calculated by using (17) in terms of the subspace angles. By comparison, the equivalence of these two methods can be clearly found in the figures. It should be noted that the deepness of gray in the bar graph implies damage level. The damage indicators increase monotonically with the inter-storey stiffness reduction, which provides the potential to estimate the damage severity. Since the damage occurs between two adjacent storeys, as shown in both figures, the damage indicators for these two storeys are a lot larger than the others.

Case II. Mutually dependent inputs: In case II, the five filtered Gaussian white noise processes are assumed to be identical. Consequently, there are strong correlations among the multiple excitations. For canceling the correlation of the mutually dependent inputs, the first step of signal processing using pre-whitening filter, as shown in Figure 2, is applied. Before calculating the damage indicators, the simulated sensor signals are decorrelated by a pre-whitening filter.
Figures 6 and 7 show the damage indicators calculated by (11) without and with pre-processing of sensor signals, respectively. It is observed that the damage indicator fails in damage identification without the pre-whitening filter, while it succeeds in identifying damage and its location when using pre-whitening filter. In this case, therefore, the pre-whitening filter can apparently improve the performance of the proposed damage indicator.

4.2. Earthquake excitation

Additionally, the proposed method is evaluated when the structure is subjected to base excitation of earthquake records, as shown in Figure 8. Considering the real application, it is impossible for a structure to be subjected to the same earthquake twice. Assume that the safe structure is subjected to the earthquake El Centro 1940, while the damaged structure is subjected to another earthquake Hachi 1968. In the simulation, the acceleration responses induced by earthquake excitation are also added by measurement noise of 10% damage level. Although the earthquake excitation case is a single-input multiple-output (SIMO) model, in numerical simulation the excitations can be approximated by five inertial forces acting on every DOF. This case is similar to the MIMO model of the ambient force case introduced above. Furthermore, the acceleration responses induced by the inertial forces are mutually correlated. Hence, in this case, the pre-whitening filter is also necessary. The damage indicators are calculated by (17) in terms of subspace angles. Figures 9 and 10 show the results without and with using the pre-whitening filter, respectively. Similar to the results in case II of Section 4.1, it is verified again that the pre-whitening filter is efficient for cancellation of correlation and helpful for damage detection.
Figure 9. Damage indicators without using pre-whitening filter subjected to earthquake excitations: (a) one-storey; (b) two-storey; (c) three-storey; (d) four-storey; and (e) five-storey damage cases.

Figure 10. Damage indicators using pre-whitening filter subjected to earthquake excitations: (a) one-storey; (b) two-storey; (c) three-storey; (d) four-storey; and (e) five-storey damage cases.
5. CONCLUSIONS

The damage indicator defined as the distance between ARMA models was proposed in this paper to identify structural damage. This cepstral distance can be calculated by two equivalent approaches, using either the poles and zeroes or the subspace angles of ARMA models. Furthermore, the proposed damage detection methodology suggested using a pre-whitening filter to cancel the correlation of sensor signals beforehand. This signal pre-processing technique overcomes the limitation that the cepstral distance can classify only the time series of SISO stochastic model or MIMO stochastic model with mutually independent inputs with the same variance. Thus, it extends the proposed damage indicator to wider areas.

The proposed methodology was tested on a five-storey shear building model by simulation. Some typical excitations, such as ambient forces and earthquake actions, have been investigated. The damage indicators, calculated by using both approaches, have efficiently identified the damages and their locations. When the excitations are mutually correlated, the pre-whitening filter can significantly improve the performance of damage indicator, especially for damage localization. Actually, the simulation cases are tested on a simple multiple DOF structural model; hence, more investigations are needed to test on complicated structures. The proposed damage indicator provides the potential to estimate the damage severity; however, for real damage quantification, the relation between the damage indicator and damage severity should be further evaluated carefully.

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